

# Adapting Fourier Analysis For Predicting Earth, Mars And Lunar Orbiting Satellite's Telemetry Behavior

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## Abstract

Equipment analog telemetry behavior is used to determine the operational behavior of equipment on-board multi-million and multi-billion dollar spacecraft. Some measurement behavior is driven by the shape of the orbit or the trajectory such as thermal measurements while others are affected by the temperature changes that must occur. During the manufacturing and test of spacecraft, there is no requirement for the builder to generate predictions for normal telemetry behavior for comparison with actual telemetry behavior to suspect on-board equipment may be operating in a manner that may render them useless requiring the backup unit to be employed. Prognostic technology uses predictive algorithms for the identification of deterministic behavior that is embedded in completely normal appearing telemetry from fully functional equipment. The predictive algorithms used to define normal behavior from which deterministic behavior is identified can be adapted to generate normal spacecraft telemetry behavior for satellites in orbit about a moon or planet or during interplanetary travel. Time-series analog engineering data (telemetry) from orbiting satellites and interplanetary spacecraft are defined by harmonic and non-harmonic influences, which shape its behavior. This paper explains how spectrum analysis can be used to understand and quantify the fundamental behavior of spacecraft analog telemetry and relate the behavior's amplitude, frequency and phase to its time-series behavior through Fourier analysis. Fourier analysis can be used by the mission operations engineering staff as a tool to confirm whether on-board spacecraft/satellite equipment is performing as expected thus decreasing risk of failure and increasing safety and mission assurance for spacecraft and satellite equipment that has never been observed from space before.

## I. Introduction

Space vehicle dynamic environmental factory acceptance testing (ATP) does not mimic the environment that spacecraft will experience while in space. Factory testing does expose the spacecraft to the worse case vibration, acoustic, shock, thermal and vacuum conditions to encourage unreliable equipment to fail if the design and manufacturing causes a defect that would cause them to fail when exposed to the environment that is present in the launch vehicle fairing getting to space and while in space. Thus, the expected analog telemetry behavior that will occur at in the environment experienced at altitude, inclination and sun angles is not available to be provided over to the mission control personnel to determine whether actual analog telemetry behavior is normal after launch or whether equipment on-board is experiencing behavior indicative of a failure.

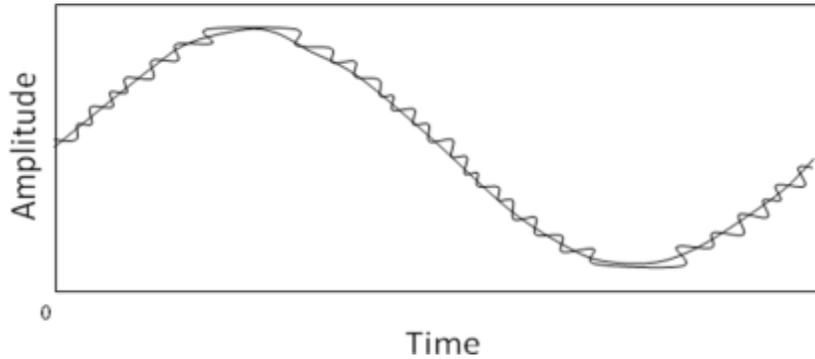
The same tools used by RF and digital signal design engineers for identify signal integrity offers new understanding for telemetry behavior from space. Analysis illustrates the harmonic properties of telemetry behavior as a function of time, amplitude, frequency and phase. Expanding spectral analysis to satellites and spacecraft illustrates their fundamental harmonic properties. This information can be used to improve vehicle reliability and define vehicle and ground station telemetry system design performance parameters and reduce risk of catastrophic satellite and spacecraft failure.

Spacecraft analog telemetry is reconstructed time-series information whose behavior is composed of many influences that are discernible using spectral analysis. Spectral analysis which relates time-series data to its

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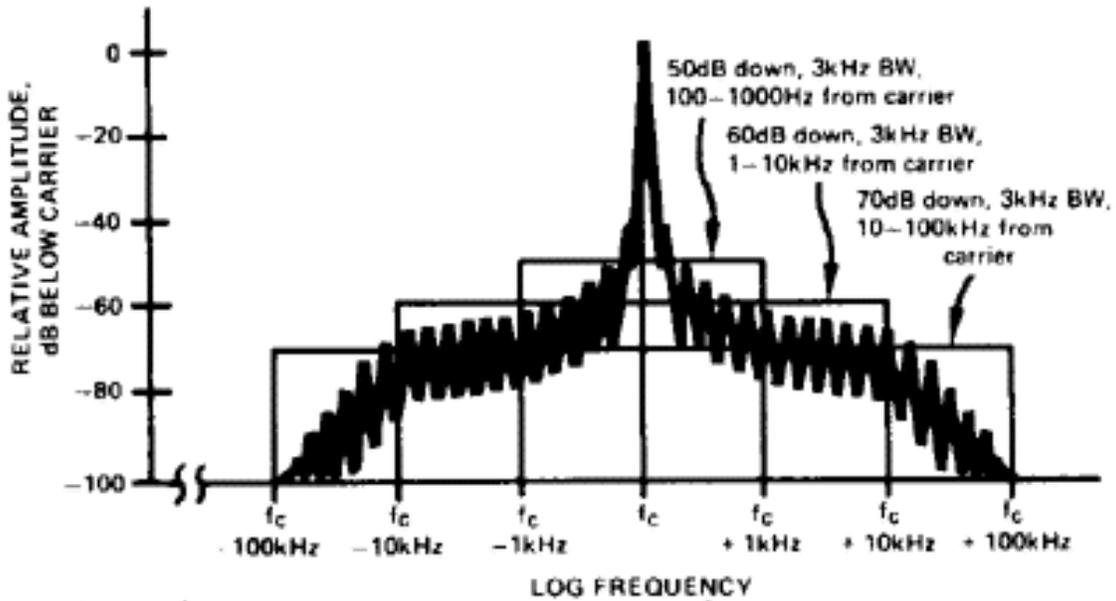
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frequency and phase components and illustrates the harmonic properties of telemetry behavior from spacecraft as a function of time, amplitude, frequency and phase and an can define and quantify normal spacecraft behavior for all applications in space. Expanding the use of spectral analysis to include the spacecraft reconstructed analog telemetry signals from spacecraft identifies all their fundamental harmonic properties so that behavior can be understood. To reduce risk of catastrophic spacecraft failure from a surprise equipment failure, spectral analysis can be used on the reconstructed analog telemetry signal to identify determine unreliable spacecraft equipment operational status and performance as well as generate spacecraft telemetry system performance requirements.



**Figure 1. Comparison of an Analog Signal and its Reconstructed Analog Signal using Digital Telemetry**

Figure 1 illustrates an original electrical or RF analog signal and its reconstitution using digital telemetry accomplished by a digital telemetry system.



**Figure 2. Power Spectral Density (PSD) for an RF Signal**

Satellite and launch vehicle reliability is around 25% infant mortality failure rate. Believing that there is no solution to the infant mortality problem for satellites and launch vehicle customers use commercial insurance companies to reduce risk of failures causing impact on financial assets and income. Spectral analysis is the decomposition of time-series electrical signals into its frequency and phase components. Spectral analysis is used in many applications to understand the electrical signal properties.

Figure 1 illustrates telemetry as reconstructed time-series analog signal, quantifiable using its amplitude, frequency and phase just as other electrical signals can be and the use mathematics developed for signal analysis and

orbital mechanics. Space engineers remain unaware of the intelligence in telemetry signal behavior, usually referred to as systemic noise.

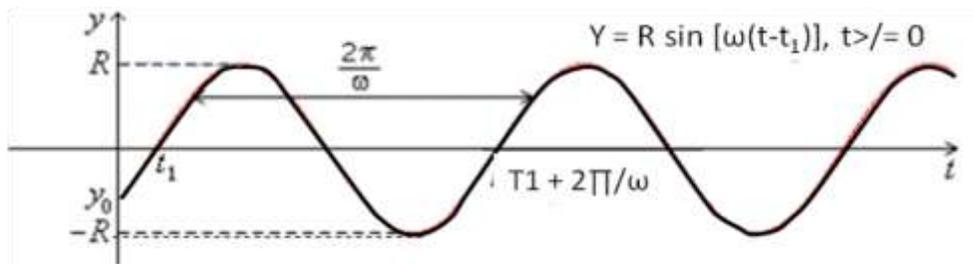
<sup>1</sup>Telemetry is a source of information that can be used to increase space vehicle reliability and safety. Telemetry has an industry reputation as being expensive, complex, unnecessary, unreliable and costly and space system purchasers such as NASA and the Air Force do not specify to the space systems builder the number of measurements and accuracy of telemetry so space system builders minimize its use. Space system builders consider the customer past request when deciding the number and accuracy of telemetry measurements to provide on a space system. NASA, INTELSAT and the Air Force ask for more telemetry than usual. First time space system customers do not have experience to recognize the need to request adequate telemetry and trust the builder to provide an adequate number of measurements who will often use only a minimal number to get through factory testing.

Telemetry Science<sup>®</sup> makes telemetry behavior understandable, reliable, quantifiable and key to space program success, and can justify the cost of adding telemetry to all on-board equipment. Using RF and digital electrical signal spectral analysis, the normal harmonic influences can be understood and leveraged as another tool for engineers to minimize risk of catastrophic spacecraft equipment failure.

## II. Virtual Analog Electronic Signal (VAES)

Telemetry originating from satellites in space, exhibit sinusoidal/harmonic behavior similar to electrical and RF signals. Until now, these properties have been relatively underutilized. Engineers responsible for the on-orbit operations and maintenance of spacecraft and satellites use telemetry to determine the spacecraft Bus equipment and payload status and operational functional performance. Telemetry is often received and recorded and made available in large quantities to engineers for evaluation often overwhelming them. Engineers have developed many software tools to automate the evaluation of large quantities of satellite telemetry. The most common tool to determine the correctness of telemetry behavior is upper and lower limits.

Telemetry in the aerospace industry from satellites and spacecraft is a reconstruction of an analog signal. It is not continuous analog data but segmented based on the sampling frequency used for each measurement. For satellites in a circular orbit, without external influences, a satellite's earth orbit is a circular or elliptical, however, the Earth's non-uniformity and influences from the sun, all other moons and planets and other influences causes both in-track and out-of-track effects. Starting with a perfect orbit circular shape, and no short or long-term influences, the behavior that a typical analog telemetry measurement creates while in orbit is that of a sinusoidal signal similar to an electrical signal. This (virtual) analog electrical signal (VAES) can be treated as an electrical signal with all the same properties and thus the mathematical tools used to quantify and understand electrical signal properties can be shared with the behavior from a VAES.



**Figure 3. A Harmonic Function.**

To quantify the behavior of electrical and RF signals, Fourier analysis is used also called harmonic analysis. <sup>2</sup>Harmonic and Fourier analysis yields the decomposition of behavior in terms of a sum of sinusoidal and cosine functions that can be recombined to obtain an approximation to the original function. Every analog or digital signal can be written as a (infinite) sum of sine and cosine functions of different frequencies; this is the idea of Fourier analysis, where trigonometric series are used to solve a variety of boundary-value problems using partial differential equations.

To convert from satellite orbit position to harmonic time series data (telemetry), trigonometric functions are used. The sine of a real number, t is given by the y-coordinate (height) of the point P in the following diagram,

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where  $t$  is the distance of the arc shown. The sin of a real number  $t$  is given by the y-coordinate (height) of the point P in the following diagram, in which  $t$  is the length of the arc shown. Figure 3 illustrates circular or elliptical motion converted to times series data. From the relationship for time and amplitude varying electrical signals, Fourier analysis uses:

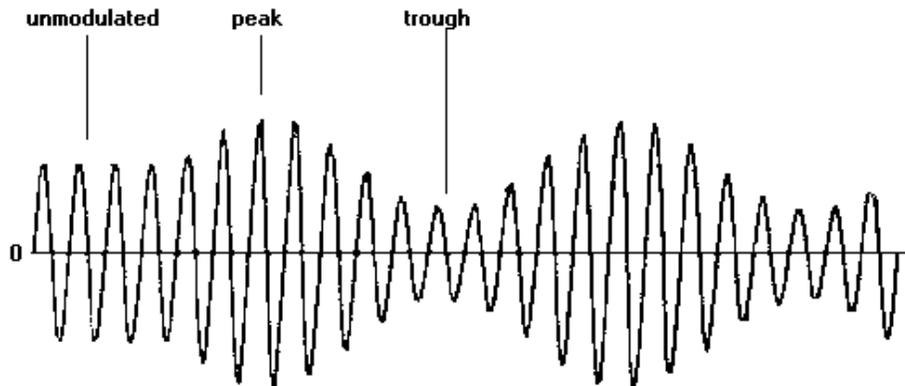
$$\text{For: } x(t) = A\sin(\omega t + \varphi)$$

$$\int x(t) = f_{\omega}(\omega)$$

$$\int f_{\omega}(\omega) = f_{\varphi}(\varphi)$$

Fourier's representation of functions as a superposition of sine's and cosines has become ubiquitous for both the analytic and numerical solution of differential equations and for the analysis and treatment of electrical and RF communication signals. Figure 4 represents time series and magnitude data, frequency and amplitude components and phase magnitude components for an analog measurement. Fourier analysis changes time-series data to frequency and phase data when the original behavior can be put into an equation form. Frequency data shows when time series-data changes which aids in identifying important values within the original time series data. Because orbiting satellites repeat their behavior every orbit, telemetry measurement behavior repeat their behavior every orbit period allowing the use of the same principals used in electrical and RF signal analysis.

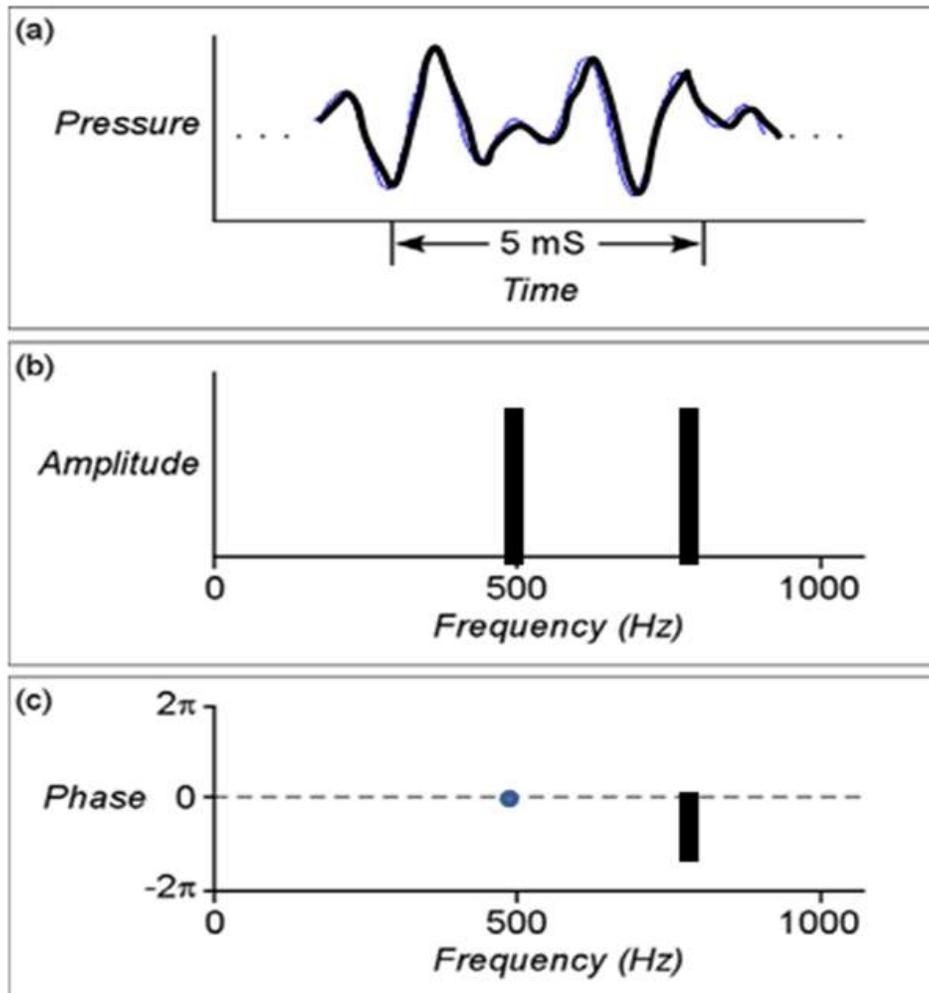
For an electrical signal, modulating the amplitude, frequency or phase provides a means of adding information. For telemetry behavior, modulating the amplitude of the VAES occurs from external harmonic influences such as the changing sun-to-orbit plane angle ( $\beta$ ). For an electrical signal, when the carrier is modulated, its amplitude goes above and below its unmodulated amplitude. The maximum percentage modulation possible is 100%. Going above this causes distortion. Distortion is bad because our electrical equipment technology cannot accurately recover information from an intentionally distorted signal. Some electrical and RF signals that are intentionally distorted can be recovered by knowing how the originally signal was distorted. Figure 4 illustrates modulated and unmodulated electrical signal.



**Figure 4. An Example of both the Amplitude Unmodulated Portion and Modulated RF or Electrical Signal**

<sup>3</sup>Modulation is a process in which a modulator changes some attribute of a higher frequency carrier signal proportional to a lower frequency message signal. A change in the message signal will produce a corresponding change in the amplitude, frequency or phase of the carrier or a change in a combination of these. A signal transmitter can then send this carrier signal through the communication medium more efficiently than the message signal alone. Finally, a receiver will demodulate the signal, recovering the original message.

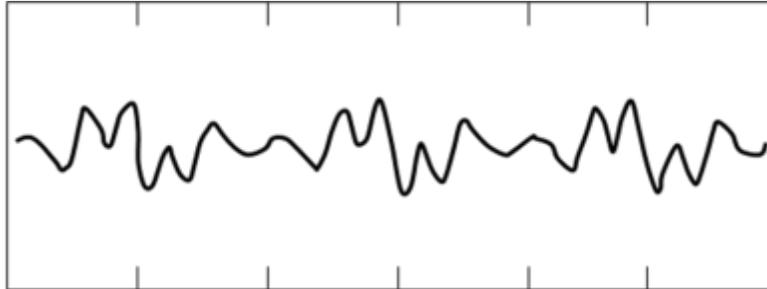
In amplitude modulation (AM), the amplitude of the carrier changes based on the amplitude of the message. The message signal rides on top of the carrier as the amplitudes of both vary with time. The frequency of the carrier, however, is much higher than the frequency of the message. This carrier frequency is the center of the 'channel,' or frequency allocation of this signal. Frequency allocations vary depending on the medium of transmission. <sup>4</sup>For broadcast transmissions, where signals are sent through the air, the government regulates frequency allocation. If the RF signal is transmitted over wire, such as in cable television, there is more freedom in the choice of carrier.



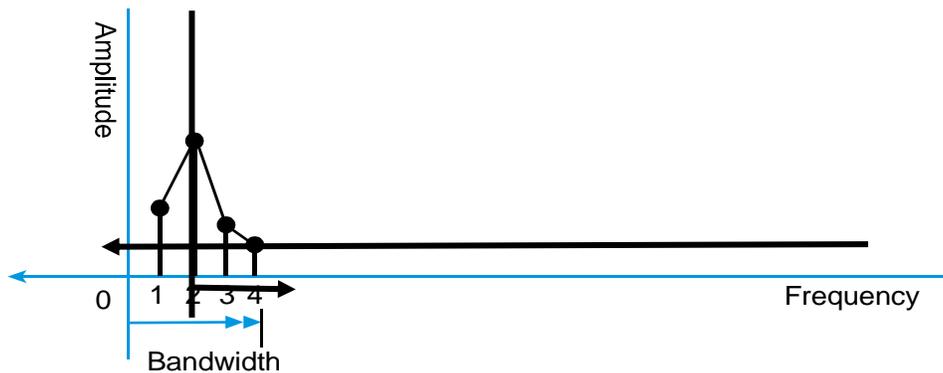
**Figure 5. An Analog Pressure Measurement's Sinusoidal Behavior (a) and its Frequency (b) and Phase (c) Spectrum**

### III. Baseband, Pass Band Signals And Amplitude Modulation

Due to their low frequency content, the information signals have a spectrum such as that in the Figure 6. There are a low frequency components and the one-sided spectrum is located near the zero frequency. The hypothetical signal in Figure 7 has four sinusoids are near 0 Hz frequency. The frequency range of this signal extends from zero to a maximum frequency of  $f_m$ . We say that this signal has a bandwidth of  $f_m$ . In the time domain, this 4-frequency component signal may look as shown in Figure 7.



**Figure 6. An Example of Time Domain Low Frequency Steady State Information Signal with Phase Modulation**



**Figure 7. The Frequency Spectrum for Figure 6.**

#### **IV. Harmonic Influences Of Satellite Telemetry**

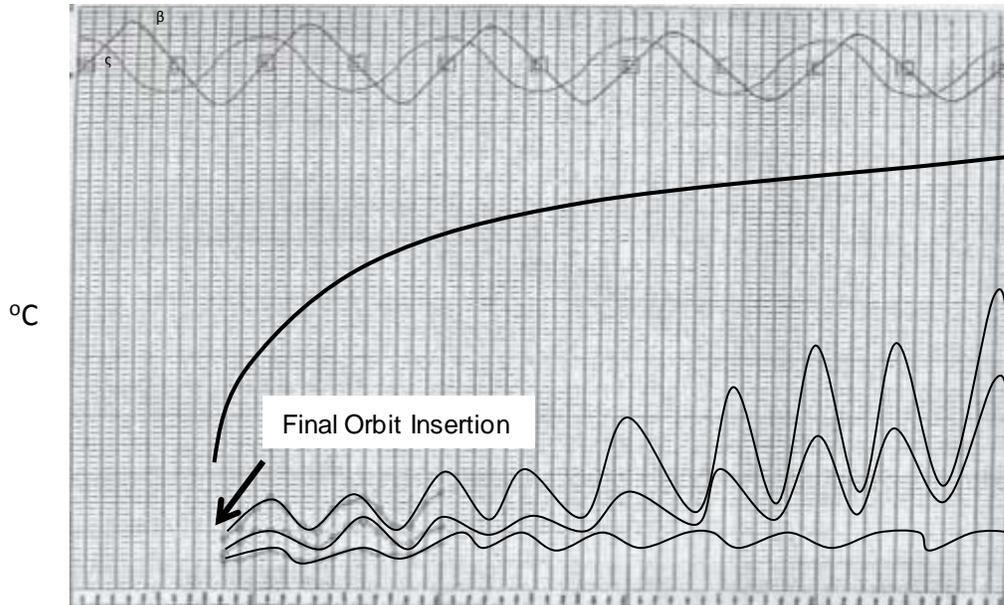
Harmonic influences of satellite VTES include (1): orbit plane drift rate caused by solar, lunar and planetary gravity forces changing sun-to-orbit plane angles ( $\beta$ ) and (2): the earth's solar constant ( $\zeta$ ) which changes ~5% peak-to-peak per year.

<sup>5</sup>The beta ( $\beta$ ) angle is the angle between the satellite orbit plane and sun vector. Beta is fixed for sun-synchronous orbits and variable for asynchronous orbit planes. Beta can vary from  $0^\circ$ , when the sun is in the orbit plane and  $90^\circ$  when the orbit plane is orthogonal to the orbit plane. For low earth circular orbits, orbit planes,  $\beta$  changes very quickly.

#### **V. Non-Harmonic Influences**

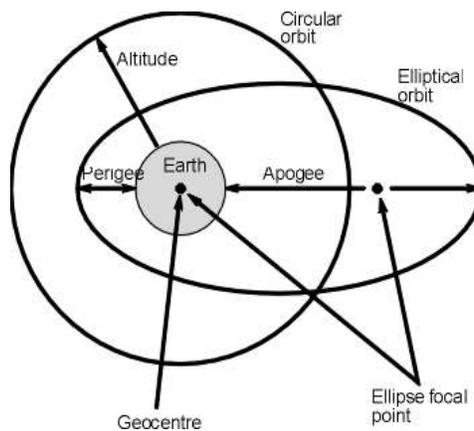
Non-harmonic influences include the location of the telemetry measurement, either internally to equipment or located in an area or region inside or outside the vehicle. Analog measurements in different quadrants will behave within well definable phase relationships based on the difference in time from exposure to solar input plus or minus a delay. The magnitude of the peak and minimum values may also differ.

Another non-harmonic influence on telemetry behavior is the change in spacecraft's thermal blanket absorptivity/emissivity ( $\alpha$ ). Satellite and spacecraft thermal blanket ability to provide insulation and protection changes in a predictable way. Thermal blankets shield the equipment from the damaging effects of solar radiation. <sup>6</sup>Without the protection of earth's atmosphere, spacecraft are exposed to the full energy spectrum of the sun, which degrades everything it radiates. When the solar radiation is not present, the equipment is exposed to the extreme low temperatures of space.



**Figure 10. The Minimum, Average And Maximum Sinusoidal Telemetry Values for the 4 Year Mission Life of a PS Satellite In a 12 Hour Orbit, Inclined  $63^{\circ}$  without Equipment Power Cycling Including Relationship between the GPS  $120^{\circ}$  Orbit Plane Beta Angle and Solar Constant and Thermal Degradation due to Solar Radiation**

## VI. Elliptical And Circular Orbits



**Figure 10. Figure Circular And Elliptical Orbits Shapes**

For a satellite in a circular orbit, its speed is constant and its altitude from the earth is fixed. A satellite in an elliptical orbit, velocity changes with its position around the orbit and its altitude changes symmetrically. In an elliptical orbit, the earth is located at one focus. The satellite's orbit will have a perigee and an apogee. <sup>7</sup>A satellite in an elliptical orbit exhibits different virtual telemetry signal behavior than from a circular orbit. In an elliptical orbit, the velocity is higher during perigee than during apogee and the time that a satellite is close to the earth is far shorter than the time it is far away from the earth. While a satellite is approaching perigee, its velocity increases. When a

satellite is moving away from the earth after passing through perigee, its velocity slows until it reached apogee and then begins to increase again after it passes apogee and is on its way back to perigee.

The virtual telemetry electronic signal for a satellite in an elliptical orbit has a fixed frequency and phase but is not symmetric in amplitude. For satellites in elliptical orbits, the earth can be much closer at perigee than for circular orbiting satellites and may influence the behavior of many measurements. Because of a significant perigee, the unbalanced gravitational forces for an elliptical orbit during perigee passes can cause its  $\Omega$ -dot to be very high. With continuously varying altitude, higher  $\Omega$ -dot, the eclipse periods may not be as symmetrical as for circular orbits. Their duration of eclipses and their frequency of eclipses are more difficult to envision.

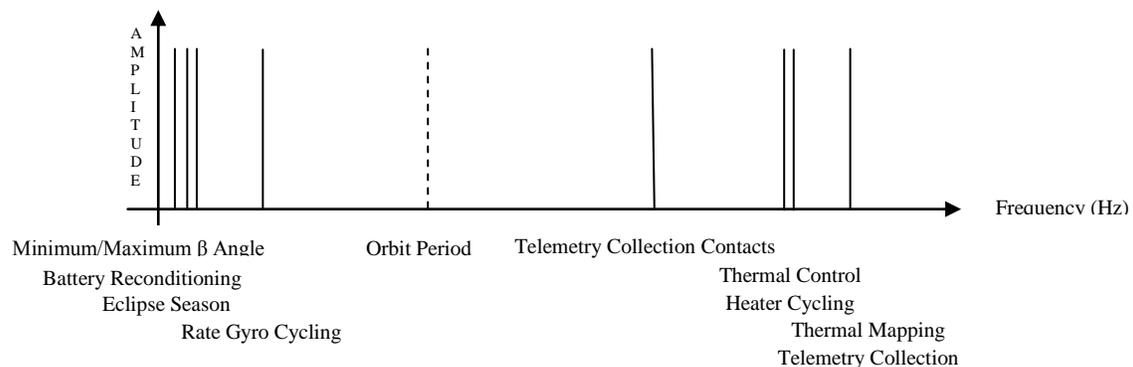
The virtual telemetry electronic signal for an elliptical orbit can be made from the positive amplitude sinusoidal function and a negative amplitude sinusoidal function. The point at which the virtual telemetry electronic signal is 0-amplitude is equal to the ratio of the semi-minor axis to the semi-major axis of the ellipse the orbital period.

## VII. Influences From Satellite Design Parameters On Virtual Telemetry Electronic Signal Behavior

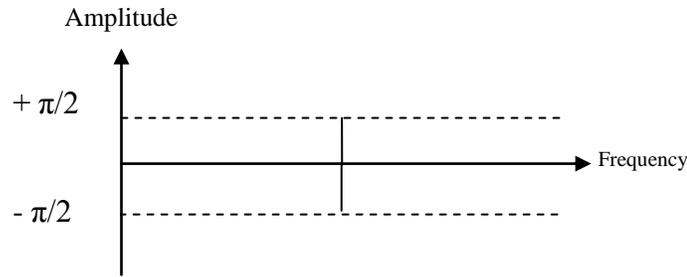
Figure 10 illustrates the long term analog telemetry measurement behavior from a measurement located inside a satellite with a 12 hour orbit for a 30 day period. Because telemetry behavior from satellites is identical in behavior of an analog electrical signal, the analysis used to understand the properties of an electrical signal, e.g. modulation, demodulation, S/N, etc. can be used. Due to the large amount of telemetry often available for analysis, only the orbital minimum, average and maximum values are used for Figure 10. A satellites regular equipment duty cycling is observable in changes in VAES behavior frequency and phase analysis and includes:

- Equipment thermal heater cycling every 3 hours ( $9.25 \times 10^{-5}$  Hz)
- Equipment thermal heater cycling every 4 hours ( $6.9 \times 10^{-5}$  Hz)
- Rate gyro cycling weekly ( $1.65 \times 10^{-7}$  Hz)
- TT&C subsystem activation every 6 hours ( $4.6 \times 10^{-5}$  Hz)
- Battery reconditioning every 5 months ( $7.7 \times 10^{-8}$  Hz)
- Eclipse season operations every 5 months for a 30 day period ( $6.4 \times 10^{-8}$  Hz)
- An increase in the frequency to 1/hour for TT&C contacts at  $0^\circ$  sun-to-orbit plane angle to determine thermal blanket emmissivity/absorptivity rate change ( $2.7 \times 10^{-4}$ )
- Minimum sun-to-orbit plane angle every 6 months ( $6.4 \times 10^{-8}$  Hz)
- Maximum sun-to-orbit ( $\beta$ ) plane angle every 6 months ( $6.4 \times 10^{-8}$  Hz)

Figure 11 illustrates the frequency spectrum for the harmonic behavior in figure 10.



**Figure 11. Frequency Spectrum (Positive Frequencies) For Boeing GPS Block I Satellite In Medium Earth Orbit With Telemetry Behavior In Figure 10.**



**Figure 8. Phase Spectrum For Figure 11**

### CONCLUSION

Fourier analysis and spectral analysis can determine normal satellite equipment telemetry behavior from space increasing the value of telemetry to ensuring satellite safety, availability and mission success. Sharing many properties as electrical signals, telemetry behavior from satellites can be another tool to quantify satellite and spacecraft equipment integrity. The intelligence added using harmonic signals to electrical and RF signals are similar to the harmonic influences that effect normal telemetry behavior and can be used to define equipment behavior. This analysis can be used by engineers to decrease risk for spacecraft owners and operators.

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